



Examination Accreditation Program
Principles for Writing Real Estate Mathematics Items
Including Examples of Problems in Items and Solutions

The content of this document was provided by Larry Outlaw in a presentation on behalf of the Examination Accreditation Committee at the ARELLO annual meeting on September 19, 2013.

The ARELLO® Examination Accreditation Program (EAP) has a set of *Item Writing Principles* that accredited examination providers are expected to follow when writing and reviewing examination items (questions). All of the EAP *Item Writing Principles* that apply to multiple-choice items in general also apply to real estate mathematics items. Because real estate mathematics items have proven to be a challenge for item writers and reviewers, this presentation focuses on a couple of general principles for all items plus principles that apply specifically to real estate mathematics items.

If you are called upon to write or review license examination mathematics items, please remember these principles.

General Rules:

- Any scenario used must be REALISTIC and appropriate.
- The incorrect answer choices should be “PLAUSIBLE” to the candidate who does not possess the requisite knowledge of the matter being tested.
- The “stem” (main body of item) must provide sufficient information not only to select the correct answer (the “key”), but also to support the plausibility of all “distractors” (incorrect answer choices).

Mathematics Item Rules

Note that all real estate mathematics items are by their nature “application” items involving a “fact situation” or “scenario.”

- The stem of a mathematics item must reflect realistic job-related activities and present an array of facts and figures from which the candidate must choose the appropriate information to use to derive the correct answer.
- Distractors must be derivable by working the problem incorrectly (e.g., based on common errors of reasoning, misconception, misinterpretation, miscalculation, selection of information, relevance of fact, or selection of partly correct options).

Common Problems with Mathematics Items

- Scenario does not describe a realistic problem commonly encountered by licensees.
- One or more of the distractors is not plausible because it cannot be derived by incorrectly using the information provided in the stem.
- The scenario is too simplistic and does not provide sufficient information in the stem to support three plausible distractors.

Examples of problematic real estate mathematics items and potential solutions follow.

Example 1 (tests calculation of depreciation in value in reverse)

A house purchased three years ago has depreciated 25% based on its current appraised value of \$215,000. What was the purchase price of the house (to the nearest thousand dollars)?

- A. \$269,000 ($.25 \times \$215,000 = \$53,750$; $\$215,000 + \$53,750 = \$268,750$)
- B. \$275,000 (cannot derive from information provided)
- C. \$287,000 **key** (see solution below)
- D. \$290,000 (cannot derive from information provided)

Solution: $100\% \text{ of PP} - 25\% \text{ of PP} = \$215,000$
 $75\% \text{ (or } .75) \times \text{PP} = \$215,000$
 $\text{PP} = \$215,000 \div .75 = \$286,667 \text{ rounded to } \$287,000$

Problems

1. *Item does not present a **realistic scenario** commonly encountered by real estate licensees. A licensee would rarely, if ever, need to use current property value and a depreciation or appreciation rate to solve for the sale price of a property at a specific time in the past; the licensee will almost always know or easily be able to obtain that figure. This item essentially tests the calculation of depreciation in value *in reverse*. It is possible that a licensee who knows the sale price of a house at a certain point in the past and knows the depreciation or appreciation rate for comparable houses since that sale may want to calculate the probable sale price of the house using that information. However, such a calculation would likely be only to corroborate the licensee's findings as to probable sale price using the sales comparison approach, which will produce a more accurate estimate than estimating depreciation in value. It is also possible that a licensee may want to know the depreciation or appreciation rate over a period of time for properties in an area, but would know the value (indicated by sale price) of the properties in the past and the current appraised value.*
2. *Aside from the key (C), only distractor A can be derived using the meager facts provided in the stem. Distractors B and D are simply "pulled out of the air." When a distractor cannot be derived by working a problem incorrectly by misusing the facts provided, candidates can easily eliminate that option even if they don't know how to perform the calculation needed. This flaw effectively makes this item a two-choice rather than a four-choice item, thereby increasing a candidate's odds of guessing the answer from 25% to 50%. Sufficient facts must be included in item stems to enable the uninformed candidate to derive each distractor by using the wrong information and/or working the problem incorrectly.*

Key point to remember about testing very simplistic mathematics processes

Many simplistic mathematics processes cannot be properly tested using the four-choice item format if the only facts provided are those minimally necessary to perform the simplistic process. Item writers and reviewers must consider both the item writing principle that requires plausible distractors and the principle that cautions against including superfluous information in item stems. To have plausible distractors in mathematics items, one of two approaches should be followed: (1) Include some information in the stem that is not needed to answer the question correctly, but is needed to provide a basis for distractors and test the candidate's ability to determine the appropriate facts to use in solving the problem, or (2) test the simplistic mathematics process in the context of a more complex process requiring more than one calculation.

Example 2 (tests use of the Gross Rent Multiplier method)

If the gross rent multiplier is \$125.50 on a 1,500 square foot house, what is the gross monthly rent if the property is worth \$130,000?

- A. \$550 (cannot derive from information provided)
- B. \$1,036 **key** ($\$130,00 \div \$125.50 = \$1,036$)
- C. \$1,894 (cannot derive from information provided)
- D. \$9,060 (cannot derive from information provided)

Problems

1. Like Example 1, *this item does not present a realistic scenario commonly encountered by real estate licensees*. In practice, a licensee never has to solve for the *existing* gross monthly rent based on a known gross rent multiplier. Generally, the GRM valuation method is used to determine the value (by appraisers) or probable sale price (by real estate licensees), in which case the gross monthly rent and the appropriate gross rent multiplier will be known. A licensee determining an appropriate rental amount to recommend may also need to research recently sold similar properties to derive an appropriate gross rent multiplier. In such case, he/she will know or obtain the sale price and the monthly rents for the similar properties. The licensee can calculate the gross rent multiplier for each and apply the derived gross rent multiplier to the appraised value (or sale price if very recently sold) of the subject property to derive an estimate of appropriate rent.
2. *Only the key can be derived using the facts provided in the stem. See comments for Example 1 for reasons this is inappropriate.*
3. *Information included provides an inappropriate clue.* Given that the size (1,500 square feet) of the house is not needed to perform the required calculation or to provide a basis for distractors, it would seem it was included to clue the candidate as to which of the widely varying answer choices is most likely correct. This makes the item even more objectionable. Given the answer choices, most candidates should be able to choose the key without even attempting to work the problem.
4. The GRM is mistakenly cited in the item as a dollar amount. This is just a multiplier and is not an actual dollar amount.

Better alternative item than Example 2 (testing use of Gross Rent Multiplier Method)

A broker manages a 1,500 square foot rental house with gross monthly income of \$1,200 and net monthly operating income of \$800. A highly similar 1500 square foot house located nearby rents for \$1,300 per month and has net monthly operating income of \$850. The comparable house just sold for \$150,000. Using the gross rent multiplier method and the data given, the best estimate of probable selling price for the subject rental house is (to the nearest thousand dollars)

- A. \$138,000 **key** ($150,000 \div 1300 = 115.38 \times 1200 = \$138,456$ or \$138,000 rounded)
- B. \$141,000 ($150,000 \div 850 = 176.47 \times 800 = \$141,176$ or \$141,000 rounded)
- C. \$146,000 ($850 \times 12 = 10,200 \div 150,000 = .068$ (6.8% cap rate); $850 + 800 = 1650 \div 2 = 825$ avg. $\times 12 = 9,900 \div .068 = \$145,588$ or \$146,000 rounded)
- D. \$150,000 (use sale price of comparable because houses are so similar)

Example 3 (tests calculation of sale price needed to provide \$X to seller)

A seller wishes to net \$200,000 after paying the listing broker a 6% commission. To accomplish this, the sale price must be at least which of the following amounts (to the nearest dollar)?

- A. \$206,274 (can't be derived from info provided)
- B. \$212,000 ($.06 \times 200,000 = 12,000 + 200,000 = 212,000$)
- C. \$212,766 key
- D. \$224,000 (can't be derived from info provided)

Solution: $100\% \text{ of SP} - 6\% \text{ of SP} = 200,000$
 $94\% (.94) \times \text{SP} = 200,000$
 $\text{SP} = 200,000 \div .94 = 212,765.95 \text{ rounded to } 212,766$

Problem

1. The point being tested is highly appropriate – virtually every seller wants to know this when setting a listing price and deciding on a sale price. However, *the stem of this item does not provide sufficient factual information to make the scenario realistic and provide a basis for three plausible distractors*. Distractor B may be derived by incorrectly calculating the needed sale price, but distractors A and D cannot be derived using information provided. To constitute a truly typical “real world” problem and provide a basis for plausible distractors, the stem should include the amount of the seller’s existing mortgage payoff and estimated closing costs the seller will be required to pay.

Better alternative item for Example 3 (testing calculation of sale price needed to net \$X to seller)

A seller wishes to net \$200,000 after paying the listing broker a 6% commission, paying off a mortgage balance of \$45,525 and paying closing costs of approximately \$1,200. To accomplish this, the sale price must be at least which of the following amounts (to the nearest dollar)?

- A. \$246,725 (omit brokerage commission)
- B. \$258,725 (miscalculate commission as \$12,000)
- C. \$261,273 (omit \$1,200 closing costs)
- D. \$262,473 key (see solution below)

Solution: $\text{SP} - 6\% \text{ of SP} - \$1,200 - \$45,225 = \$200,000$
 $.94 \times \text{SP} = \$200,000 + \$45,525 + \$1,200$
 $\text{SP} = \$246,725 \div .94 = \$262,473.40 \text{ or } \$262,473 \text{ rounded}$

Example 4 (tests calculation of property value using income capitalization method)

An investment property returns 11% of its value or \$33,000 annually. What is the value of the property?

- A. \$220,000 (cannot derive from information provided)
- B. \$300,000 **key** ($33,000 \div 11\%$ (or $.11$) = 300,000)
- C. \$330,000 (cannot derive from information provided)
- D. \$363,000 ($33,000 \times .11 = 3630$; misplace decimal for 363,000)

Problems

1. *Item is unrealistic and uses inappropriate terminology.* Without saying so, the item implies that the candidate is to use the income capitalization formula to solve the problem, although the cited income is not stated to be the “net operating income,” which is needed to use the income capitalization formula. In practice, when the value of the subject property is not known, it is not possible to know the capitalization rate for the subject property. Rather, it is necessary to determine an appropriate capitalization rate based on analysis of data on recently sold comparable properties and the rate determined would be a “projected” rate instead of the actual rate referred to in the stem.
2. *Item is poorly conceived and written.* The answer can be derived by anyone without any knowledge of the income capitalization method. This can be done in two ways: (1) Multiply each of the options by $.11$ to see which result provides the \$33,000 income figure given in the stem; or (2) divide \$33,000 by each of the options and multiply by 100 to find which result equals 11%.
3. *Only the key (B) and arguably distractor D can be derived by misusing the facts provided in the stem.* There is no basis for anyone to mistakenly derive distractors A and C.

Better alternative item than Example 4 (testing use of income capitalization formula to calculate an investment property’s probable selling price)

An investment property managed by a licensee has gross annual income of \$46,000 with \$10,000 in annual operating expenses and \$3,000 in expenditures for capital improvements during the past year. Analysis of data on recently sold comparable properties indicates an appropriate capitalization rate would be 11%. Which of the following should the licensee estimate to be the probable selling price?

- A. \$300,000 **key** ($46,000 - 10,000 - 3,000 = 33,000$ in NOI $\div .11 = 300,000$)
- B. \$327,273 ($36,000 \div .11 = 327,273$)
- C. \$363,000 ($33,000 \times 11 = 363,000$)
- D. \$418,182 ($46,000 \div .11 = 418,182$)

NOTE: Note in the better alternates in examples 2 and 4 that the term “probable selling price” rather than “value” is used to describe the amount that the licensee would derive when performing a *broker price opinion (BPO)* or a *comparative market analysis* because a number of states have passed laws authorizing real estate licensees to perform BPOs or CMAs *for a fee* but only if they describe their estimate of a property’s worth as the “**probable selling or leasing price**” rather than the “**value.**” Under these laws, any reference to “value” in an estimate makes the estimate an “appraisal” that can only be performed by a licensed/certified appraiser.